Double-Λ hypernuclei in the relativistic mean-field theory

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We study the properties of double-Λ hypernuclei in the relativistic mean-field theory, which has been successfully used for the description of stable and unstable nuclei. With the meson-hyperon couplings determined by the experimental binding energies of single-Λ hypernuclei, we present a self-consistent calculation of double-Λ hypernuclei in the relativistic mean-field theory, and discuss the influence of hyperons on the nuclear core. The contribution of two mesons with dominant strange quark components (scalar σ^* and vector ϕ) to the $\Lambda\Lambda$ binding energy of double-Λ hypernuclei is examined.

§1. Introduction

The study of hypernuclei has been attracting great interest of nuclear physicists due to its important role in providing information about hyperon-nucleon and hyperon-hyperon interactions. Such information is crucial for understanding the properties of multi-strange systems and neutron stars.[1)–6)] The most extensively studied hypernuclear system is the single-Λ hypernucleus which consists of a Λ particle coupled to the nuclear core. There exist many experimental data for various single-Λ hypernuclei over almost the whole mass table.[7)–9)] However, only three double-Λ hypernuclei, ${}^{6}_{\Lambda\Lambda}$He, ${}^{10}_{\Lambda\Lambda}$Be, and ${}^{13}_{\Lambda\Lambda}$B, have been identified experimentally.[10)–13)] A recent observation of the double-Λ hypernucleus ${}^{6}_{\Lambda\Lambda}$He, called the Nagara event,[14)] has had a significant impact on strangeness nuclear physics. The Nagara event provides unambiguous identification of ${}^{6}_{\Lambda\Lambda}$He production with precise $\Lambda\Lambda$ binding energy value $B_{\Lambda\Lambda} = 7.25 \pm 0.19^{+0.18}_{-0.11}$ MeV, which suggests that the effective $\Lambda\Lambda$ interaction should be considerably weaker ($\triangle B_{\Lambda\Lambda} = B_{\Lambda\Lambda}({}^{6}_{\Lambda\Lambda}\mathrm{He}) - 2\mathrm{B}_{\Lambda}({}^{5}_{\Lambda}\mathrm{He}) \approx$ 1 MeV) than that deduced from the earlier measurement ($\triangle B_{\Lambda\Lambda} \approx 5$ MeV).[11)] The weak $\Lambda\Lambda$ interaction suggested by the Nagara event has triggered great interest in theoretical studies of double-Λ hypernuclei by using various approaches, such as cluster models, Faddeev calculations, and coupled channel methods.[15)–18)] It has also been used to examine the properties of strange hadronic matter.[5),19)–21)]

The purpose of this paper is to present a self-consistent calculation of double-Λ hypernuclei in a wide range of mass number A within the framework of relativistic mean-field theory (RMF), and also to examine possible contributions of the two strange mesons σ^* and ϕ to the $\Lambda\Lambda$ binding energy. In principle, exact few-body

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calculations are more appropriate for light hypernuclei, but they are not available for heavy systems. Therefore, we adopt the RMF theory to study the mass number dependence of various quantities in double-Λ hypernuclei. By performing the self-consistent calculation in the RMF theory, we hope to constrain the meson-hyperon couplings by reproducing available experimental data, especially the Nagara data, which suggests a weak $\Lambda\Lambda$ interaction. It is well known that these coupling constants play important roles in multi-strange systems and neutron stars.[2)–6),19)–22)]

The RMF theory has been quite successfully used for the description of nuclear matter and finite nuclei, including unstable nuclei and deformed nuclei.[23)–25)] It has also been applied to predict the equation of state of dense matter for the use in supernovae and neutron stars.[3),26)] There are many works using the RMF theory for the study of single-Λ hypernuclei,[27)–32)] in which the meson-hyperon couplings were determined by fitting the experimental Λ binding energies of single-Λ hypernuclei. The Λ binding energy is obtained experimentally by $B_\Lambda\left({}^{A}_{\Lambda}Z\right) = B\left({}^{A}_{\Lambda}Z\right) - B\left({}^{A-1}Z\right)$,[9)] which might be different from the Λ single-particle energy due to the core polarization effect. It has been shown that the rearrangement energy, which represents the core polarization effect, decreases with increasing mass number A, and is negligible in most cases.[27),33)] Therefore, the experimental Λ binding energy is usually considered as the Λ single-particle energy, and is used to be compared with the calculated Λ single-particle energy in various models. In the present work, we determine the meson-hyperon couplings by reproducing the experimental Λ binding energies of single-Λ hypernuclei, then apply the RMF model to study the properties of double-Λ hypernuclei. We discuss also the core polarization effect in double-Λ hypernuclei. For hypernuclear systems containing more than one hyperon, it has been discussed in early works[22),34)] that there might be contributions from the two mesons with dominant strange quark components, scalar σ^* and vector ϕ, which couple exclusively to hyperons. We examine these contributions to the $\Lambda\Lambda$ binding energies of double-Λ hypernuclei. In Ref. 34), the $\Lambda\Lambda$ correlation effect has been estimated by solving a Schrodinger equation with spherical harmonic oscillator potentials provided by the nuclear core and an effective $\Lambda\Lambda$ interaction. However, the approximations used in their treatment have violated the self-consistency between the potentials and the baryon wave functions. In the present work, we would rather keep the self-consistency of the calculation than take into account the $\Lambda\Lambda$ correlation seriously.

The outline of this paper is as follows. In Sec. 2 we briefly explain the formulation of hypernuclear systems in the RMF theory. The model parameters and the method to determine the meson-hyperon couplings are discussed in Sec. 3. The calculated results of double-Λ hypernuclei are presented in Sec. 4. Sec. 5 is devoted to a summary.

§2. Model

In the RMF theory, baryons interact via the exchange of mesons. The baryons involved in the present work are nucleons and Λ hyperons, while the exchanged mesons consist of isoscalar scalar and vector mesons (σ and ω) and isovector vector

meson (ρ). When it is applied to a strange nuclear system containing more than one hyperon, such as a double-Λ hypernucleus, we consider two models. The first one (model 1) is a simple application of the RMF theory containing only usual mesons (σ, ω, and ρ), and the second one (model 2) incorporates two additional mesons (scalar σ^* and vector ϕ) which couple exclusively to hyperons. The effective Lagrangian, after the mean-field approximation is applied, may be written as

$$\begin{aligned}\mathcal{L} = {} & \bar{\psi}\left[i\gamma_\mu\partial^\mu - M_N - g_\sigma\sigma - g_\omega\gamma^0\omega - g_\rho\gamma^0\tau_3\rho - e\gamma^0\frac{1+\tau_3}{2}A\right]\psi \\ & +\bar{\psi}_\Lambda\left[i\gamma_\mu\partial^\mu - M_\Lambda - g_\sigma^\Lambda\sigma - g_{\sigma^*}^\Lambda\sigma^* - g_\omega^\Lambda\gamma^0\omega - g_\phi^\Lambda\gamma^0\phi + \frac{f_\omega^\Lambda}{2M_\Lambda}\sigma^{0i}\partial_i\omega\right]\psi_\Lambda \\ & -\frac{1}{2}(\bigtriangledown\sigma)^2 - \frac{1}{2}m_\sigma^2\sigma^2 - \frac{1}{3}g_2\sigma^3 - \frac{1}{4}g_3\sigma^4 - \frac{1}{2}(\bigtriangledown\sigma^*)^2 - \frac{1}{2}m_{\sigma^*}^2\sigma^{*2} \\ & +\frac{1}{2}(\bigtriangledown\omega)^2 + \frac{1}{2}m_\omega^2\omega^2 + \frac{1}{4}c_3\omega^4 + \frac{1}{2}(\bigtriangledown\phi)^2 + \frac{1}{2}m_\phi^2\phi^2, \\ & +\frac{1}{2}(\bigtriangledown\rho)^2 + \frac{1}{2}m_\rho^2\rho^2 + \frac{1}{2}(\bigtriangledown A)^2,\end{aligned} \tag{2·1}$$

where ψ and ψ_Λ are the Dirac spinors for nucleons and Λ hyperons. The mean-field values of the exchanged mesons are denoted by σ, ω, ρ, σ^*, and ϕ, respectively. We note that σ^* and ϕ have no contribution to a single-Λ hypernucleus because these mesons are assumed to be exchanged exclusively between two hyperons due to the Okubo-Zweig-Iizuka (OZI) rule. A is the electromagnetic field which couples only to the protons. The Λ hyperon is a charge neutral and isoscalar particle so that it does not couple to ρ and A. We include the non-linear terms for σ and ω mesons, which are introduced to reproduce properties of finite nuclei and the feature of the nucleon self-energy obtained in the relativistic Bruckner-Hartree-Fock theory.[35] We taken into account the tensor coupling between ω and Λ as suggested in previous works.[28)–31] It is known that the tensor coupling is capable of resolving the problem of the small spin-orbit interaction in single-Λ hypernuclei, but it has negligible influence on the $\Lambda\Lambda$ binding energy of double-Λ hypernuclei.[36]

The Dirac equations for nucleons and Λ hyperons have the following form:

$$\left[i\gamma_\mu\partial^\mu - M_N - g_\sigma\sigma - g_\omega\gamma^0\omega - g_\rho\gamma^0\tau_3\rho - e\gamma^0\frac{1+\tau_3}{2}A\right]\psi = 0, \tag{2·2}$$

$$\left[i\gamma_\mu\partial^\mu - M_\Lambda - g_\sigma^\Lambda\sigma - g_{\sigma^*}^\Lambda\sigma^* - g_\omega^\Lambda\gamma^0\omega - g_\phi^\Lambda\gamma^0\phi + \frac{f_\omega^\Lambda}{2M_\Lambda}\sigma^{0i}\partial_i\omega\right]\psi_\Lambda = 0. \tag{2·3}$$

The equations of motion for mesons are given by

$$-\Delta\sigma + m_\sigma^2\sigma + g_2\sigma^2 + g_3\sigma^3 = -g_\sigma\rho_s - g_\sigma^\Lambda\rho_s^\Lambda, \tag{2·4}$$

$$-\Delta\omega + m_\omega^2\omega + c_3\omega^3 = g_\omega\rho_v + g_\omega^\Lambda\rho_v^\Lambda + \frac{f_\omega^\Lambda}{2M_\Lambda}\rho_T^\Lambda, \tag{2·5}$$

$$-\Delta\rho + m_\rho^2\rho = g_\rho\rho_3, \tag{2·6}$$

$$-\Delta\sigma^* + m_{\sigma^*}^2\sigma^* = -g_{\sigma^*}^\Lambda\rho_s^\Lambda, \tag{2·7}$$

$$-\Delta\phi + m_\phi^2\phi = g_\phi^\Lambda \rho_v^\Lambda, \qquad (2\cdot8)$$

$$-\Delta A = e\rho_p, \qquad (2\cdot9)$$

where ρ_s (ρ_s^Λ), ρ_v (ρ_v^Λ), ρ_T^Λ, ρ_3, and ρ_p are the scalar, vector, tensor, third component of isovector, and proton densities, respectively. The preceding coupled equations should be solved self-consistently for various hypernuclear systems. We restrict our study to the spherical case, and take into account the pairing contribution for open shell nuclei by using the BCS theory.

There are several recipes for performing the center-of-mass correction in the RMF theory, which have been discussed and compared in Ref.[37] It is well known that the relative contribution of the center-of-mass correction to the total binding energy is very large for light nuclei, therefore, we should treat the center-of-mass correction seriously. In the present calculation, we take the microscopic scheme suggested in Ref,[37]

$$E_{\rm c.m.} = \frac{\langle F|\,\hat{\mathbf{P}}^2_{\rm total}\,|F\rangle}{2M_{\rm total}}, \qquad (2\cdot10)$$

where $M_{\rm total} = \sum M_B = nM_\Lambda + (A-n)M_N$ is the total mass of the system containing n Λ hyperons, while $\hat{\mathbf{P}}_{\rm total} = \sum \hat{\mathbf{P}}_B$ is the total momentum operator. The expectation value of the square of the total momentum operator is calculated from the actual wave function of the ground state of the hypernucleus. We perform self-consistent calculations for the hypernuclear systems containing one or two Λ hyperons.

§3. Parameters

For the parameters of the nucleonic sector, we employ two successful parameter sets TM1 and NL-SH as listed in Table I, which could provide excellent descriptions for nuclear matter and finite nuclei including unstable nuclei.[35),38)] The TM1 set includes non-linear terms of both σ and ω mesons, while the NL-SH set contains only non-linear σ terms. As for the meson-hyperon couplings, it is well known that the properties of single-Λ hypernuclei are very sensitive to the ratios of the meson-hyperon couplings to the meson-nucleon couplings, $R_\sigma = g_\sigma^\Lambda/g_\sigma$ and $R_\omega = g_\omega^\Lambda/g_\omega$.[32] We take the naive quark model value for the relative ω coupling as $R_\omega = 2/3$, while the relative σ coupling is constrained by fitting to the experimental Λ binding energies of single-Λ hypernuclei. It is shown in Fig. 1 that $R_\sigma = 0.621$ could fairly reproduce the experimental values of Λ binding energies in a wide range of mass number A for both TM1 and NL-SH parameter sets. In the present calculation, we adopt the quark model value of the tensor coupling, $f_\omega^\Lambda = -g_\omega^\Lambda$.[28] The inclusion of tensor coupling is important to produce small spin-orbit spitting of single-Λ hypernuclei.[28)–30)]

After the meson-hyperon couplings are determined by the experimental Λ binding energies of single-Λ hypernuclei, the RMF theory can be applied straightforwardly to predict the properties of double-Λ hypernuclei. This is referred to as model 1 in the present calculation, in which the exchanged mesons are limited to the usual

mesons σ, ω, and ρ. There are some arguments that the two mesons σ^* and ϕ may give essential contributions to multi-hyperon systems.[22),34)] In this work, we incorporate these two additional mesons in model 2, and investigate the dependence of the contributions on the coupling constants. We take the experimental meson masses as $m_{\sigma^*} = 980$ MeV and $m_\phi = 1020$ MeV. For the ϕ coupling, we adopt the quark model relationship $R_\phi = g_\phi^\Lambda / g_\omega = -\sqrt{2}/3$, while the relative σ^* coupling, $R_{\sigma^*} = g_{\sigma^*}^\Lambda / g_\sigma$, is taken as an adjustable parameter in model 2. The two mesons σ^* and ϕ were originally introduced to obtain strong $\Lambda\Lambda$ attraction ($\triangle B_{\Lambda\Lambda} \approx 5$ MeV) deduced from the earlier measurement.[22)] However, it is now believed that the $\Lambda\Lambda$ interaction is much weaker ($\triangle B_{\Lambda\Lambda} \approx 1$ MeV) as suggested by the striking Nagara event. We first determine the relative coupling R_{σ^*} by reproducing the experimental value $\triangle B_{\Lambda\Lambda} \approx 1$ MeV deduced from the Nagara event, then change the value of R_{σ^*} in some range to examine the dependence of the results on this parameter.

§4. Results

We calculate several double-Λ hypernuclei including light, medium, and heavy systems within the framework of RMF theory by using two successful parameter sets TM1 and NL-SH as listed in Table I. For the meson-hyperon couplings, we adopt $R_\sigma = 0.621$, $R_\omega = 2/3$, and $f_\omega^\Lambda = -g_\omega^\Lambda$, which could give reasonable descriptions of single-Λ hypernuclei in a wide range of mass number A as shown in Fig. 1. We present in Table II the $\Lambda\Lambda$ binding energy $B_{\Lambda\Lambda}$, which is obtained for the hypernucleus ${}^{A}_{\Lambda\Lambda}Z$ by

$$\begin{aligned} B_{\Lambda\Lambda}\left({}^{A}_{\Lambda\Lambda}Z\right) &= B\left({}^{A}_{\Lambda\Lambda}Z\right) - B\left({}^{A-2}Z\right) \\ &= M\left({}^{A-2}Z\right) - M\left({}^{A}_{\Lambda\Lambda}Z\right) + 2M_\Lambda. \end{aligned} \tag{4·1}$$

We also list the quantity $\triangle B_{\Lambda\Lambda}$, which is called the $\Lambda\Lambda$ bond energy, defined by

$$\triangle B_{\Lambda\Lambda}\left({}^{A}_{\Lambda\Lambda}Z\right) = B_{\Lambda\Lambda}({}^{A}_{\Lambda\Lambda}Z) - 2B_\Lambda({}^{A-1}_{\Lambda}Z). \tag{4·2}$$

The limited experimental data of light double-Λ hypernuclei are listed for comparison, while the calculated results of heavy systems are considered as the predictions in the present models. It is seen that $B_{\Lambda\Lambda}$ increases with increasing mass number A, while $\triangle B_{\Lambda\Lambda}$ decreases. We show in this table the calculated results with both TM1 and NL-SH parameter sets by using models 1 and 2. In model 1, the exchanged mesons are limited to the usual mesons σ, ω, and ρ, whose coupling constants are determined by the experimental data of single-Λ hypernuclei. Therefore, no more adjustable parameters exist when model 1 is used to the calculation of double-Λ hypernuclei. In model 2, two additional mesons σ^* and ϕ are included, so we should determine their couplings properly. Here the quark model value $R_\phi = -\sqrt{2}/3$ is adopted, while $R_{\sigma^*} = 0.57$ (TM1) and $R_{\sigma^*} = 0.56$ (NL-SH) are constrained by the experimental value $\triangle B_{\Lambda\Lambda}({}^{6}_{\Lambda\Lambda}\mathrm{He}) \approx 1$ MeV deduced from the Nagara event.[14)] Without adjusting any parameter in model 1, the calculated $\triangle B_{\Lambda\Lambda}({}^{6}_{\Lambda\Lambda}\mathrm{He})$ is very close to the experimental value. It is found that the results of model 2 are almost identical to those of model 1. This is because the contributions from σ^* and ϕ mesons in model

2 are mostly cancelled with each other, so there is no obvious difference between the two models for the calculations of double-Λ hypernuclei. However, there might be some noticeable contributions from σ^* and ϕ mesons when model 2 is applied to the studies of multi-strange systems and neutron stars.

It is very interesting to discuss the influence of Λ hyperons on the nuclear core, which is known as the core polarization effect.[27),33),34)] We compare the properties of the nucleus with those of the nuclear core in single or double-Λ hypernuclei. It is known that nucleons in the core would be affected by the additional Λ hyperons in hypernuclei. The so-called rearrangement energy E_R quantifies the core polarization effect, which represents the change of nuclear core binding energies caused by the presence of Λ, given by

$$E_R = \sum_{i=1}^{n} \epsilon_{\Lambda}^{i} - B_{n\Lambda}({}^{A}_{n\Lambda}Z), \qquad (4{\cdot}3)$$

where ϵ_{Λ}^{i} is the absolute value of Λ single-particle energy, and n denotes the number of Λ hyperons in the hypernucleus. We present in Table III the calculated results of several nuclei together with corresponding single and double-Λ hypernuclei in the RMF model with TM1 parameter set, and the results of double-Λ hypernuclei are obtained by using model 1. It is seen that the rearrangement energy E_R decreases rapidly with increasing mass number A, and is usually negligible in comparison with the binding energy except for very light systems. For example, the rearrangement energy of ${}^{42}_{\Lambda\Lambda}$Ca, which contains two $1s$ Λ hyperons coupled to the nuclear core ^{40}Ca, is 0.29 MeV, and this value is less than 1% of the $\Lambda\Lambda$ binding energy ($B_{\Lambda\Lambda} =$ 38.15 MeV, see Table II). But, the rearrangement energy of ${}^{6}_{\Lambda\Lambda}$He ($E_R = 3.54$ MeV) is rather large in comparison with the $\Lambda\Lambda$ binding energy ($B_{\Lambda\Lambda} = 5.52$ MeV). We note that $B_{\Lambda\Lambda} = 2\epsilon_{\Lambda} - E_R$ for double-Λ hypernuclei. Therefore, the rearrangement energy for a light system may significantly contribute to the binding energy, and is not negligible. In Ref. 34), the rearrangement energy of the double-Λ hypernucleus has been assumed to be twice the one of the single-Λ hypernucleus. It is seen from Table III that this assumption is satisfied within an error of $\sim 10\%$ except for ${}^{210}_{\Lambda\Lambda}$Pb which has a negligible rearrangement energy. On the other hand, the single-particle energies of neutrons and protons in hypernuclei are also affected by the presence of Λ hyperons. We list the single-particle energies of neutrons at $1s$ states ($\epsilon_n(1s)$) and rms radii (r_{Λ}, r_n, and r_p) in Table III. It is shown that the change of the rms radii of neutrons and protons is quite small. Usually the radius of a particle increases as the strength of its potential decreases. The Λ potential is much shallower than the nucleon potential, so the rms radius of Λ at $1s$ state should be rather larger than the one of nucleon at $1s$ state, but it may be smaller than the radius of nucleon at higher state in heavy systems. Hence, the rms radius of Λ is larger than the total rms radius of neutrons or protons in a light system, while it should be smaller than those in heavy systems. It is obvious that the influence of Λ on the nuclear core is significantly weakened with the increase of mass number A.

For very light hypernuclei like ${}^{6}_{\Lambda\Lambda}$He, the center-of-mass correction provides a significant contribution to the total binding energy, and there exist considerable

differences between different schemes to take into account the center-of-mass correction. For instance, the value of the center-of-mass correction of ${}^{6}_{\Lambda\Lambda}$He obtained from Eq. (2·10) is 10.25 MeV in model 1 with NL-SH parameter set, while it turns out to be 16.92 MeV with the usual approximation

$$E_{\mathrm{c.m.}} = \frac{3}{4} \cdot 41\ A^{-1/3}\ (\mathrm{MeV}), \tag{4·4}$$

so it is rather large in comparison with the total binding energy of 32.66 MeV obtained in this case. The influence of the center-of-mass correction on $\triangle B_{\Lambda\Lambda}$ is visible for light hypernuclei. The simple approximation Eq. (4·4) yields $\triangle B_{\Lambda\Lambda}({}^{6}_{\Lambda\Lambda}\mathrm{He}) =$ 0.73 MeV in agreement with the value presented in Table 2 for parametrization P5 of Ref. 36), while the microscopic scheme Eq. (2·10) results in $\triangle B_{\Lambda\Lambda}({}^{6}_{\Lambda\Lambda}\mathrm{He}) =$ 1.08 MeV as shown in Table II. Therefore, we emphasize that the center-of-mass correction plays an important role in light systems, but its influences on hypernuclear properties and the differences between different schemes decrease rapidly with increasing mass number A. In this work, the calculated results of $B_{\Lambda}({}^{5}_{\Lambda}\mathrm{He})$ and $B_{\Lambda\Lambda}({}^{6}_{\Lambda\Lambda}\mathrm{He})$ are underestimated in comparison with the experimental values. This might be due to the limited reliability of the RMF approach for light systems.

So far it is difficult to determine a reliable σ^* coupling by scarce and various difficult measurements of double-Λ hypernuclei. In order to investigate the possible contributions from the two mesons σ^* and ϕ, we plot $\triangle B_{\Lambda\Lambda}$ of several double-Λ hypernuclei as a function of R_{σ^*} in Fig. 2. Some large values of R_{σ^*} were used in previous studies[22),34)] in order to obtain strong $\Lambda\Lambda$ interaction as $\triangle B_{\Lambda\Lambda} \approx 5$ MeV deduced from the earlier measurement.[10),11)] This has been changed by the Nagara event,[14)] which leads to a small $\Lambda\Lambda$ bond energy $\triangle B_{\Lambda\Lambda} \approx 1$ MeV. It is found that the contributions from σ^* and ϕ for heavy hypernuclei are much smaller than those for light systems. For instance, at $R_{\sigma^*} \approx 0.75$ the light hypernuclei ${}^{6}_{\Lambda\Lambda}$He gets about 4 MeV extra binding energy in model 2 compared with the result in model 1, while it is less than 1 MeV for heavy hypernuclei like ${}^{210}_{\Lambda\Lambda}$Pb. For each species of the double-Λ hypernuclei, $\triangle B_{\Lambda\Lambda}$ shows a decrease tendency with decreasing R_{σ^*}.

There are many discussions in the literature about possible mechanisms to soften the equation of state (EOS) of neutron star matter, e.q., by hyperons, kaon condensates, or even quark phases.[2)–5),39),40)] It is well known that the hyperon-hyperon interactions play important roles in determining the abundance of hyperons at high densities. The Nagara event[14)] has suggested a much weaker $\Lambda\Lambda$ interaction than the one used in early calculations.[2)] We would like to discuss the neutron star properties changed due to the weak $\Lambda\Lambda$ interaction. Here we focus on the contribution of Λ hyperons only, since we have rather poor knowledge of other hyperon interactions. It is shown in Ref. 29) that the inclusion of Λ hyperons could considerably soften the EOS at high densities. We compare the results of neutron star properties, which are calculated in model 2 with TM1 parameter set. The maximum mass of neutron stars using $R_{\sigma^*} = 0.57$, which is constrained by $\triangle B_{\Lambda\Lambda} \approx 1$ MeV, is about 0.1 $M_{\odot}$ larger than the one using $R_{\sigma^*} = 0.75$ determined by $\triangle B_{\Lambda\Lambda} \approx 5$ MeV. It is obvious that the weak $\Lambda\Lambda$ interaction suggested by the Nagara event leads to a slightly stiffer EOS than the strong $\Lambda\Lambda$ interaction.

§5. Summary

In summary, we have performed the self-consistent calculations for double-Λ hypernuclei in a wide range of mass number A within the framework of RMF theory. We have adopted the parameter sets TM1 and NL-SH, which could provide excellent descriptions for finite nuclei and single-Λ hypernuclei. We have studied the properties of double-Λ hypernuclei using two models. Model 1 is the RMF theory containing the exchanged mesons σ, ω, and ρ. Model 2 incorporates two additional mesons σ^* and ϕ, which couple exclusively to hyperons. The results of model 2 are definitely dependent on the couplings of σ^* and ϕ. With the couplings constrained by $\triangle B_{\Lambda\Lambda} \approx 1$ MeV deduced from the Nagara event, the results of model 2 are almost identical to those of model 1.

The influence of Λ hyperons on the nuclear core has been investigated by comparing various properties of the nucleus with those of the nuclear core in single or double-Λ hypernuclei. We found that the rearrangement energy decreases rapidly with increasing mass number A, and could be neglected in most cases. But for very light system like ${}^{6}_{\Lambda\Lambda}$He, the rearrangement energy may be rather large in comparison with the $\Lambda\Lambda$ binding energy, and is not negligible. The single-particle energies of neutrons in hypernuclei are also affected by the presence of Λ hyperons.

We have examined the possible contributions from the two strange mesons σ^* and ϕ. They could give either positive or negative contributions to the $\Lambda\Lambda$ binding energies, which depend on the coupling constants used in the calculation. It is found that the contributions from σ^* and ϕ to $B_{\Lambda\Lambda}$ for heavy hypernuclei are much smaller than those for light systems. So far it is difficult to determine a reliable σ^* coupling by the scarce and various difficult measurements of double-Λ hypernuclei. It is necessary and important to get more and better experimental data on double-Λ hypernuclei so that theoretical models can be checked and extended to further study.

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Table I. The parameter sets TM1[35] and NL-SH.[38] The masses are given in MeV.

	M	m_σ	m_ω	m_ρ	g_σ	g_ω	g_ρ	g_2 (fm^{-1})	g_3	c_3
TM1	938.0	511.198	783.0	770.0	10.0289	12.6139	4.6322	-7.2325	0.6183	71.3075
NL-SH	939.0	526.059	783.0	763.0	10.444	12.945	4.383	-6.9099	-15.8337	0.0

Table II. $B_{\Lambda\Lambda}$ and $\triangle B_{\Lambda\Lambda}$ of double-Λ hypernuclei. The calculated results of models 1 and 2 are denoted by 1 and 2, respectively. The available experimental data are taken from Refs. 10)–14).

	$B_{\Lambda\Lambda}$	TM1		NL-SH		$\triangle B_{\Lambda\Lambda}$	TM1		NL-SH	
	exp.	1	2	1	2	exp.	1	2	1	2
${}^{6}_{\Lambda\Lambda}$He	7.25 ± 0.2	5.52	5.48	4.75	4.68	1.0 ± 0.2	1.07	1.03	1.08	1.01
${}^{10}_{\Lambda\Lambda}$Be	17.7 ± 0.4	16.34	16.28	16.03	15.94	4.3 ± 0.4	0.37	0.31	0.38	0.29
	14.6 ± 0.4					1.2 ± 0.4				
	8.5 ± 0.7					-4.9 ± 0.7				
${}^{13}_{\Lambda\Lambda}$B	27.5 ± 0.7	22.14	22.07	22.65	22.52	4.8 ± 0.7	0.26	0.19	0.33	0.21
${}^{18}_{\Lambda\Lambda}$O		25.89	25.85	25.30	25.23		0.14	0.10	0.14	0.07
${}^{42}_{\Lambda\Lambda}$Ca		38.15	38.13	37.90	37.86		0.04	0.02	0.04	0.00
${}^{92}_{\Lambda\Lambda}$Zr		47.11	47.10	47.73	47.71		0.03	0.02	0.04	0.02
${}^{210}_{\Lambda\Lambda}$Pb		52.19	52.19	53.03	53.02		0.03	0.02	0.02	0.02

Table III. Comparison of the energies (in MeV) and radii (in fm) of single and double-Λ hypernuclei with those of normal nuclei. The results are obtained with TM1 parameter set, and the model 1 is used for the calculation of double-Λ hypernuclei. B_{total} and B_{core} represent the total binding energies and the binding energies of the nuclear core. E_R is the rearrangement energy given in Eq. (4·3). $\epsilon_\Lambda(1s)$ and $\epsilon_n(1s)$ are the absolute values of single-particle energies for Λ and neutron at 1s states. The rms radii of Λ, neutron, and proton are denoted by r_Λ, r_n, and r_p, respectively.

	B_{total}	B_{core}	E_{R}	$\epsilon_\Lambda(1s)$	$\epsilon_n(1s)$	r_Λ	r_n	r_p
^{4}He	28.19	28.19			17.75		1.90	1.92
${}^{5}_{\Lambda}$He	30.42	26.32	1.87	4.10	19.35	2.77	1.90	1.91
${}^{6}_{\Lambda\Lambda}$He	33.71	24.65	3.54	4.53	20.90	2.74	1.89	1.90
^{16}O	128.73	128.73			40.67		2.56	2.58
${}^{17}_{\Lambda}$O	141.61	128.36	0.37	13.24	41.42	2.48	2.56	2.59
${}^{18}_{\Lambda\Lambda}$O	154.62	127.90	0.83	13.36	42.18	2.49	2.57	2.59
^{40}Ca	344.35	344.35			51.58		3.32	3.36
${}^{41}_{\Lambda}$Ca	363.40	344.22	0.13	19.18	52.04	2.76	3.32	3.36
${}^{42}_{\Lambda\Lambda}$Ca	382.49	344.06	0.29	19.22	52.51	2.77	3.32	3.37
^{208}Pb	1638.48	1638.48			55.89		5.75	5.48
${}^{209}_{\Lambda}$Pb	1664.56	1638.45	0.03	26.11	56.07	4.09	5.75	5.48
${}^{210}_{\Lambda\Lambda}$Pb	1690.67	1638.39	0.09	26.14	56.26	4.09	5.75	5.48

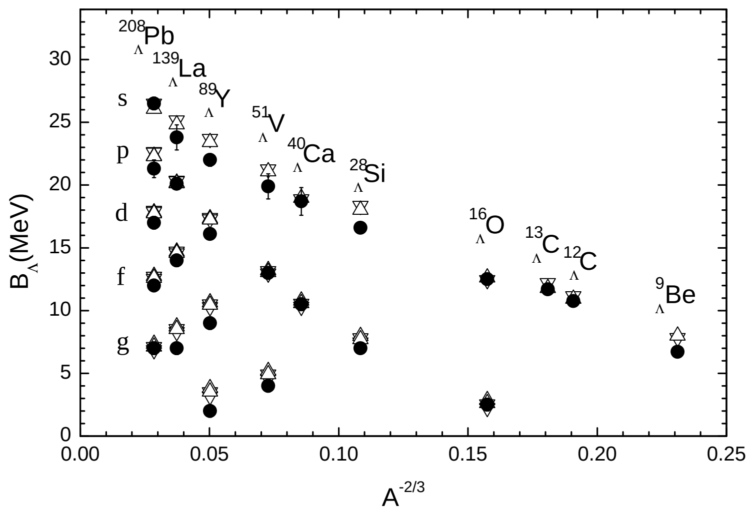

Fig. 1. The Λ binding energies in single-Λ hypernuclei. The solid circles represent the experimental data with errors taken from Refs. 7)–9). The open up and down triangles are the results in the RMF model with the parameter sets TM1 and NL-SH, respectively.

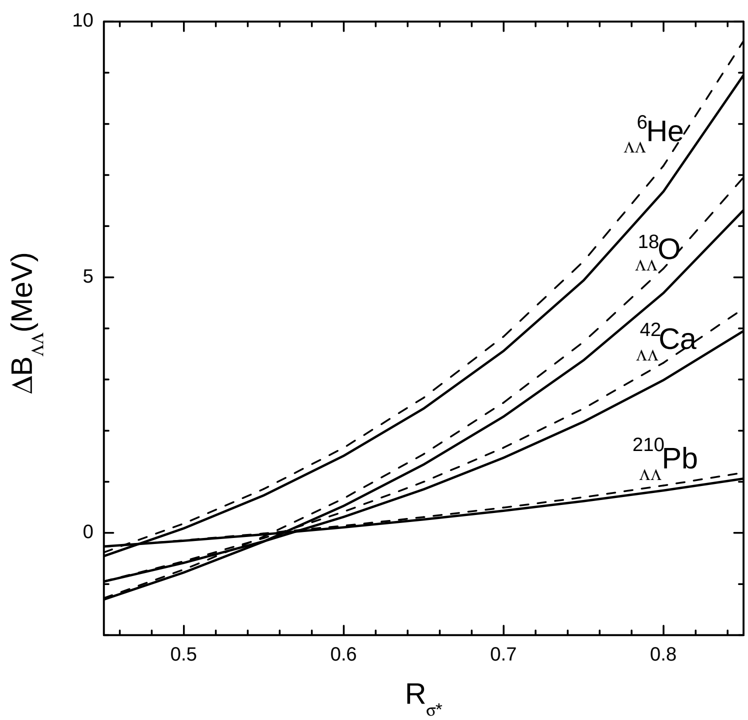

Fig. 2. $\triangle B_{\Lambda\Lambda}$ as a function of R_{σ^*} for several double-Λ hypernuclei. The solid lines represent the results in the RMF model with TM1 parameter set, while those with NL-SH set are shown by dashed curves.